

Syllabus for MATH 754  
Infinite-dimensional Lie Algebras and Applications  
Spring 2019

1. ABOUT THE COURSE

This course will be a detailed introduction into the structure and representation theory of some of the most important infinite-dimensional Lie algebras: Heisenberg algebras, Kac-Moody algebras, and Virasoro algebra.

Major topics to be covered:

- Heisenberg algebra, Virasoro algebra, and affine  $\widehat{\mathfrak{g}}$  as universal central extensions
- Representations of Heisenberg algebra, Virasoro algebra, affine  $\widehat{\mathfrak{sl}}_n$  via Lie algebras  $\mathfrak{gl}_\infty, \mathfrak{a}_\infty$ , and application to integrable systems
- Boson-fermion correspondence: vertex operator construction and Schur polynomials
- Feigin-Fuchs-Kac determinant formula for Virasoro and the region of unitarity
- The Sugawara construction and the Goddard-Kent-Olive construction
- Structure and representation theory of Kac-Moody algebras
- The Weyl-Kac character formula and the Kac-Macdonald identities
- Shapovalov-Jantzen-Kac-Kazhdan determinant formula for Kac-Moody algebras

2. LECTURES

Location: DL 431

Time: TTh 1:00–2:30pm

Instructor: Sasha Tsymbaliuk

Email: [oleksandr.tsymbaliuk@yale.edu](mailto:oleksandr.tsymbaliuk@yale.edu)

Office: LOM 219-C

Office hours: TTh 3:00–4:00pm

3. REFERENCES

The material of this course is based on:

- Book “*Bombay lectures on highest weight representations of infinite dimensional Lie algebras*” by V. Kac and A. Raina, 2nd edition, 2013.
- Expository paper “*Representations of contragredient Lie algebras and the Kac-Macdonald identities*” by B. Feigin and A. Zelevinsky, 1971 (to be distributed in the class).
- Book “*Infinite dimensional Lie algebras*” by V. Kac, 1983.

4. REQUIREMENTS

To pass the course it will be required to solve homework assignments, which will be assigned every Thursday and due the following Thursday.